

PERFECT ROMAN DOMINATION IN REGULAR GRAPHS

*Michael A. Henning and William F. Klostermeyer**

A perfect Roman dominating function on a graph G is a function $f: V(G) \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u with $f(u) = 0$ is adjacent to exactly one vertex v for which $f(v) = 2$. The weight of a perfect Roman dominating function f is the sum of the weights of the vertices. The perfect Roman domination number of G , denoted $\gamma_R^p(G)$, is the minimum weight of a perfect Roman dominating function in G . We show that if G is a cubic graph on n vertices, then $\gamma_R^p(G) \leq \frac{3}{4}n$, and this bound is best possible. Further, we show that if G is a k -regular graph on n vertices with k at least 4, then $\gamma_R^p(G) \leq \left(\frac{k^2+k+3}{k^2+3k+1}\right)n$.

1. Introduction

Let $G = (V, E)$ be an undirected graph with vertex set V and edge set E . A *neighbor* of a vertex v in G is a vertex adjacent to v . The *open neighborhood* of the vertex v in G is the set of all neighbors of v in G ; that is, $N_G(v) = \{u \in V \mid uv \in E\}$. The *closed neighborhood* of v in G is $N_G[v] = \{v\} \cup N_G(v)$. A *dominating set* of G is a set $D \subseteq V$ such that every vertex outside D has a neighbor inside D . The minimum cardinality amongst all dominating sets of G is the *domination number*, denoted as $\gamma(G)$. A thorough treatise on dominating sets can be found in [7].

A *perfect dominating set* in G is a set $S \subseteq V$ such that for all $v \in V$, $|N[v] \cap S| = 1$. Perfect dominating sets and several variations on perfect domination have received much attention in the literature; for example, see some discussion in [7] or the survey in [11].

*Corresponding author. William F. Klostermeyer

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A *Roman dominating function* of a graph G , abbreviated *RD-function*, is a function $f: V(G) \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u with $f(u) = 0$ is adjacent to at least one vertex v for which $f(v) = 2$. The *weight* of a vertex v is its value, $f(v)$, assigned to it under f . The *weight*, $w(f)$, of f is the sum, $\sum_{u \in V(G)} f(u)$, of the weights of the vertices. The *Roman domination number*, denoted $\gamma_R(G)$, is the minimum weight of an RD-function in G ; that is,

$$\gamma_R(G) = \min\{w(f) \mid f \text{ is an RD-function in } G\}.$$

Roman domination was first studied in depth in a graph theory setting in [3], after its initial introduction in the series of papers [13, 14, 15, 16]. Upper bounds for Roman domination were considered in, for example, [2].

A *perfect Roman dominating function* of a graph G , abbreviated *PRD-function*, is a function $f: V(G) \rightarrow \{0, 1, 2\}$ satisfying the condition that every vertex u with $f(u) = 0$ is adjacent to exactly one vertex v for which $f(v) = 2$. The *perfect Roman domination number*, denoted $\gamma_R^p(G)$, is the minimum weight of an PRD-function in G ; that is,

$$\gamma_R^p(G) = \min\{w(f) \mid f \text{ is an PRD-function in } G\}.$$

The function that assigns the weight 1 to every vertex in a graph G is a PRD-function of G , implying that every graph G of order n has a PRD-function and $\gamma_R^p(G) \leq n$. The concept of perfect Roman domination was introduced and first studied in [8], where it is shown that if T is a tree on $n \geq 3$ vertices, then $\gamma_R^p(G) \leq \frac{4}{5}n$. It is easy to see that this bound is sharp, as $\gamma_R^p(P_5) = 4$. The authors of [8] pose the question of whether $\gamma_R^p(G) \leq \frac{4}{5}n$ for all graphs G with at least three vertices. In this paper, we answer that question in the affirmative for k -regular graphs with $k \in \{2, 3, 4\}$, as well as for k -regular graphs of girth at least 7 and $k \in \{5, 6, 7\}$. We also show that if G is a k -regular graph on n vertices with $k \geq 4$, then $\gamma_R^p(G) \leq \left(\frac{k^2+k+3}{k^2+3k+1}\right)n$.

We note that a different notion of perfection in Roman domination was considered in [12]. In that paper, the authors study Roman dominating functions in which the vertices of weight 1 and 2 induce an independent set. Another related variant of Roman domination in which each vertex of weight 0 must be adjacent to at least two vertices weighted 2 or one vertex weighted 3 is explored in [1]; the vertices with weight 1 must also be adjacent to at least one vertex with weight 2 or 3, though the authors of that paper show that no weight 1 vertices are ever needed.

2. Notation and Terminology

For notation and graph theory terminology, we generally follow [10]. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The *order* of G is denoted by $n(G) = |V(G)|$, and the *size* of G by $m(G) = |E(G)|$. We denote the *degree* of a vertex v in G by $d_G(v)$, or simply by $d(v)$ if the graph G is clear from the context.

If every vertex in G has degree r , then G is called an r -regular graph. A 3-regular graph is commonly referred to as a *cubic graph* in the literature. The length of a smallest cycle in a graph G (containing a cycle) is the *girth* of G , denoted by $g(G)$.

The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest (u, v) -path in G . The maximum distance among all pairs of vertices of G is the *diameter* of G , denoted by $\text{diam}(G)$. We say that G is a *diameter-2 graph* if $\text{diam}(G) = 2$. The *distance* $d_G(v, S)$ between a vertex v and a set S of vertices in a graph G is the minimum distance from v to a vertex of S in G .

For a vertex v in a graph G and an integer $i \geq 1$, let $N_i(v)$ denote the set of all vertices at distance exactly i from v in G . In particular, note that $N_1(v)$ is the neighborhood, $N(v)$, of v . Further, let $N_i[v]$ denote the set of all vertices within distance i from v in G . In particular, $N_1[v]$ is the closed neighborhood, $N_G[v]$, of v . Generalizing this concept, let S be a subset of vertices in a graph G . The *open neighborhood* of S is the set $N_G(S) = \bigcup_{v \in S} N_G(v)$, and the *closed neighborhood* of S is the set $N_G[S] = N_G(S) \cup S$. For $i \geq 1$, the i^{th} *boundary* of S , denoted by $\partial_i(S)$, is the set of all vertices at distance i from the set S . We also call the 1^{st} boundary of S simply the *boundary* of S , denoted by $\partial(S)$. Thus, $\partial(S) = \partial_1(S) = N_G[S] \setminus S$ is the set of vertices outside S that have a neighbor in S . Further, we let $N_i[S]$ be the set of vertices within distance i in G from the set S . In particular, note that $N_1[S] = N_G[S]$ and $N_2[S] = S \cup \partial_1(S) \cup \partial_2(S)$.

A set S of vertices in a graph G is a *packing* in G if the closed neighborhoods of vertices in S are pairwise disjoint. Equivalently, S is a packing in G if the vertices in S are pairwise at distance at least 3 apart in G . We remark that a packing is sometimes called a 2-packing in the literature. The *packing number* of G , denoted by $\rho(G)$, is the maximum cardinality of a packing in G .

A set S of vertices in a graph G is *independent* if no two vertices in S are adjacent. The *independence number* of G , denoted by $\alpha(G)$, is the maximum cardinality of an independent set of vertices in G .

The *square* of a graph G , denoted G^2 , is the graph with $V(G^2) = V(G)$ and $uv \in E(G^2)$ if and only if $1 \leq d_G(u, v) \leq 2$. We remark that the square of a graph G is also called its 2^{nd} power in the literature.

For subsets X and Y of vertices of a graph G , denote the set of edges that join a vertex of X and a vertex of Y by $[X, Y]$. Thus, $|[X, Y]|$ is the number of edges with one end in X and the other end in Y . We also use the standard notation $[k] = \{1, \dots, k\}$.

3. Known Results

Let \mathcal{T} be the family of all trees T whose vertex set can be partitioned into sets, each inducing a path P_5 on five vertices, such that the subgraph induced by the central vertices of these paths is connected (by ‘‘central vertex’’ of P_5 , we mean the unique vertex in P_5 that is neither a leaf, nor adjacent to a leaf). The following

result is proven in [8].

Theorem 1. ([8]) *If T is a tree of order $n \geq 3$, then $\gamma_R^p(T) \leq \frac{4}{5}n$, with equality if and only if $T \in \mathcal{T}$.*

We shall need the following result due to Favaron [6].

Lemma 2. ([6]) *If G is a connected cubic graph of order n different from the Petersen graph, then $\rho(G) \geq \frac{n}{8}$.*

We next present some known results on independence in graphs. We shall need the following result shown in [9].

Theorem 3. ([9]) *If G is a graph of order n and p is an integer, such that (A) below holds, then $\alpha(G) \geq 2n/p$.*

(A): *For every clique X in G there exists a vertex $x \in X$, such that $d(x) < p - |X|$.*

We remark that Theorem 3 strengthens a result of Fajtlowicz [4, 5] on the independence number of a graph given its maximum degree and maximum clique size. As a consequence of Theorem 3, we obtain Fajtlowicz's result.

Corollary 4. ([4]) *If G is a graph of order n containing no clique of size q , then $\alpha(G) \geq 2n/(\Delta(G) + q)$.*

We note that an independent set in the square G^2 of a graph G is a packing in G ; the converse is also true. Hence, we have the following observation.

Observation 5. *If G is a graph, then $\rho(G) = \alpha(G^2)$.*

4. 2-Regular Graphs

In this section, we establish an upper bound on the perfect Roman domination number of a 2-regular graph.

Corollary 6. *For $n \geq 3$, $\gamma_R(C_n) \leq \frac{4}{5}n$, with equality if and only if $n = 5$.*

Proof. For $n \geq 3$, let $G \cong C_n$ and let G be given by $v_1v_2 \dots v_nv_1$. Let T be a spanning tree of G . Renaming vertices if necessary, we may assume that T is obtained from G by deleting the edge v_1v_2 ; that is, $T = G - v_1v_2$ and $T \cong P_n$. Let f be a PRD-function of T of minimum weight, and so $w(f) = \gamma_R^p(T)$. By Theorem 1, $\gamma_R^p(T) \leq \frac{4}{5}n$. If a leaf of T has weight 2 under f , then, by the minimality of f , its neighbor has weight 0 under f . In this case, re-assigning the weight 1 to both this leaf and its neighbor produces a new minimum PRD-function of T . Hence, we may choose the function f so that neither leaf has weight 2 under f . Thus, $f(v_1) \in \{0, 1\}$ and $f(v_n) \in \{0, 1\}$. But then f is also a PRD-function of G , and so

$\gamma_R^p(G) \leq \gamma_R^p(T) \leq \frac{4}{5}n$. Suppose further that $\gamma_R^p(G) = \frac{4}{5}n$. In this case, $\gamma_R^p(T) = \frac{4}{5}n$, and so, by Theorem 1, the tree $T \in \mathcal{T}$. However, the only path that belong to the family \mathcal{T} is the path P_5 , implying that $n = 5$ and $G \cong C_5$. \square

5. 3-Regular Graphs

In this section, we establish an upper bound on the perfect Roman domination number of a 3-regular graph. By linearity, we may assume from this point onwards that the graph under consideration is connected, for otherwise we apply the result to each component of the graph.

Theorem 7. *If G is a cubic graph of order n , then $\gamma_R^p(G) \leq \frac{3}{4}n$, and this bound is best possible.*

Proof. Let G be a cubic graph of order n . Suppose firstly that G is the Petersen graph, and let v be an arbitrary vertex of G . We note that in this case $n = 10$. Let f be the function that assigns the weight 1 to the vertex v , the weight 2 to each neighbor of v , and the weight 0 to the remaining six vertices. The resulting function f is a PRD-function of G , and so $\gamma_R^p(G) \leq w(f) = 7 = (3n - 2)/4 < 3n/4$. Hence, we may assume that G is different from the Petersen graph, for otherwise the desired result follows.

Let S be a maximum packing in G , and so $|S| = \rho(G)$. By Lemma 2, $|S| \geq n/8$. Recall that $\partial_2(S)$ is the set of vertices at distance 2 from the set S . Since S is a maximum packing, every vertex of G is within distance 2 from some vertex of S , and so $\partial_2(S) = V(G) \setminus N[S]$. Let f be the function that assigns the weight 2 to each vertex of S , the weight 0 to each vertex of $\partial(S)$, and the weight 1 to each vertex of $\partial_2(S)$. The function f is a PRD-function of G , and so

$$\begin{aligned} \gamma_R^p(G) &\leq w(f) \\ &= 2|S| + |\partial_2(S)| \\ &= 2|S| + (n - |S| - |\partial(S)|) \\ &= 2|S| + (n - 4|S|) \\ &= n - 2|S| \\ &\leq n - 2 \cdot \frac{n}{8} \\ &= \frac{3}{4}n. \end{aligned}$$

This completes the proof of the upper bound of Theorem 7. That this upper bound is best possible may be seen by taking, for example, any one of the two non-planar cubic graphs of order eight, shown in Figure 1(a) and 1(b). Both of these cubic graphs have perfect Roman domination number 6 and therefore achieve the $\frac{3}{4}$ -bound of Theorem 7. \square

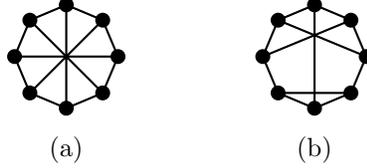


Figure 1: The two non-planar cubic graphs of order eight.

6. General Regular Graphs

In this section, we establish an upper bound on the perfect Roman domination number of a k -regular graph for all $k \geq 4$. We shall prove the following result, where recall that we may assume that the graph under consideration is connected.

Theorem 8. *If G is a k -regular graph of order n with $k \geq 4$, then $\gamma_R^p(G) \leq \left(\frac{k^2+k+3}{k^2+3k+1}\right)n$.*

Proof. Let G be a k -regular graph of order n . Let S be a maximum packing in G , so that $|S| = \rho(G)$. Since S is a maximum packing, every vertex of G is within distance 2 from some vertex of S , and so $\partial_2(S) = V(G) \setminus N[S]$. We consider two cases.

$$\text{Case 1. } |S| \geq \frac{2n}{k^2+3k+1}.$$

Let f be the function that assigns to each vertex of S the weight 2, to each vertex of $\partial(S)$ the weight 0, and to each vertex of $\partial_2(S)$ the weight 1. The function f is a PRD-function of G , and so

$$\begin{aligned} \gamma_R^p(G) &\leq w(f) \\ &= 2|S| + |\partial_2(S)| \\ &= 2|S| + (n - |S| - |\partial(S)|) \\ &= 2|S| + (n - |S| - k|S|) \\ &= n - (k-1)|S| \\ &\leq n - \left(\frac{2(k-1)}{k^2+3k+1}\right)n \\ &= \left(\frac{k^2+k+3}{k^2+3k+1}\right)n. \end{aligned}$$

$$\text{Case 2. } |S| \leq \frac{2n}{k^2+3k+1}.$$

We note that in this case,

$$\begin{aligned} |\partial_2(S)| &= n - |S| - |\partial(S)| \\ &\geq \frac{1}{2}(k^2 + 3k + 1)|S| - |S| - k|S| \\ &= \left(\frac{k^2+k-1}{2}\right)|S|. \end{aligned}$$

We now count the edges in G between $\partial(S)$ and $\partial_2(S)$. We note that every vertex in $\partial_2(S)$ has a neighbor in $\partial(S)$. Let A be the set of vertices in $\partial_2(S)$ that have at least two neighbors in $\partial(S)$, and let B be the remaining set of vertices in $\partial_2(S)$ that have exactly one neighbor in $\partial(S)$. Thus, $|A| + |B| = |\partial_2(S)|$ and

$$\begin{aligned} |[\partial(S), \partial_2(S)]| &\geq 2|A| + |B| \\ &= 2|A| + (|\partial_2(S)| - |A|) \\ &= |A| + |\partial_2(S)| \\ &\geq |A| + \left(\frac{k^2+k-1}{2}\right) |S|. \end{aligned}$$

On the other hand, since G is k -regular, every vertex in $\partial(S)$ has at most $k-1$ neighbors in $\partial_2(S)$, and so

$$|[\partial(S), \partial_2(S)]| \leq (k-1)|\partial(S)| = k(k-1)|S|.$$

Thus,

$$|A| + \left(\frac{k^2+k-1}{2}\right) |S| \leq |[\partial(S), \partial_2(S)]| \leq k(k-1)|S|,$$

implying that

$$|A| \leq \left(\frac{k^2-3k+1}{2}\right) |S|.$$

Let g be the function that assigns to each vertex of $A \cup S$ the weight 1, to each vertex of $\partial(S)$ the weight 2, and to each vertex of B the weight 1. The function g is a PRD-function of G , and so

$$\begin{aligned} \gamma_R^p(G) &\leq w(g) \\ &= |A| + |S| + 2|\partial(S)| \\ &\leq \left(\frac{k^2-3k+1}{2}\right) |S| + |S| + 2k|S| \\ &= \left(\frac{k^2+k+3}{2}\right) |S| \\ &\leq \left(\frac{k^2+k+3}{2}\right) \left(\frac{2}{k^2+3k+1}\right) n \\ &= \left(\frac{k^2+k+3}{k^2+3k+1}\right) n. \end{aligned}$$

In both Case 1 and Case 2, we have $\gamma_R^p(G) \leq \left(\frac{k^2+k+3}{k^2+3k+1}\right) n$. This completes the proof of Theorem 10. \square

We establish next an upper bound on the perfect Roman domination number of a k -regular graph with girth at least 7 for all $k \geq 3$. For this purpose, we shall need the following lemma.

Lemma 9. *If G is a k -regular graph of order n with $k \geq 3$ and with girth at least 7, then*

$$\rho(G) \geq \frac{2n}{k^2+k+2}.$$

Proof. Let G be a k -regular graph, $k > 1$, of order $n \geq 7$ with girth at least 7. Let x be an arbitrary vertex of G , and let X be an arbitrary clique in G^2 containing the vertex x . Let $N_G(x) = \{x_1, x_2, \dots, x_k\}$. For $i \in [k]$, let $X_i = N_G[x_i] \setminus \{x\}$ and let $Y_i = N_G(x_i) \setminus \{x\}$. Since G has girth at least 7, $X_i \cap X_j = \emptyset$ for $1 \leq i < j \leq k$. Further, every vertex in Y_i is at distance at least 3 in G from every vertex in Y_j for $1 \leq i < j \leq k$. Thus, either $X \subseteq N_G[x]$ or $X \subseteq N_G[x_i]$ for some $i \in [k]$. In both cases, $|X| \leq k + 1$. Hence, letting $p = k^2 + k + 2$, we note that $p - |X| \geq (k^2 + k + 2) - (k + 1) = k^2 + 1 > k^2 = d_{G^2}(x)$. Since x is an arbitrary vertex of G^2 and X is an arbitrary clique in G^2 containing x , condition (A) in the statement of Theorem 3 holds with $p = k^2 + k + 2$. Hence, by Observation 5 and Theorem 3, $\rho(G) = \alpha(G^2) \geq 2n/p = 2n/(k^2 + k + 2)$. \square

We are now in a position to prove the following result.

Theorem 10. *If G is a k -regular graph of order n with $k \geq 3$ and with $g(G) \geq 7$, then*

$$\gamma_R^p(G) \leq \left(\frac{k^2 - k + 4}{k^2 + k + 2} \right) n.$$

Proof. Let G be a k -regular graph of order n . Let S be a maximum packing in G , so that $|S| = \rho(G)$. By Lemma 9, $|S| \geq 2n/(k^2 + k + 2)$. Since S is a maximum packing, every vertex of G is within distance 2 from some vertex of S , and so $\partial_2(S) = V(G) \setminus N[S]$. Let f be the function that assigns to each vertex of S the weight 2, to each vertex of $\partial(S)$ the weight 0, and to each vertex of $\partial_2(S)$ the weight 1. The function f is a PRD-function of G , and so

$$\begin{aligned} \gamma_R^p(G) &\leq w(f) \\ &= 2|S| + |\partial_2(S)| \\ &= 2|S| + (n - |S| - |\partial(S)|) \\ &= 2|S| + (n - (k + 1)|S|) \\ &= n - (k - 1)|S| \\ &\leq n - \frac{2(k-1)}{k^2+k+2} \cdot n \\ &= \left(\frac{k^2 - k + 4}{k^2 + k + 2} \right) n. \quad \square \end{aligned}$$

As an immediate consequence of Theorem 10, we have the following result.

Corollary 11. *If G is a k -regular graph of order n with $k \in \{5, 6, 7\}$ and with girth at least 7, then $\gamma_R^p(G) \leq \frac{4}{5}n$.*

7. Closing Remarks

We close with the following problems that we have yet to settle.

Problem 1. For each $k \geq 2$, determine the smallest possible constant c_k such that every k -regular graph G of order n satisfies $\gamma_R^p(G) \leq c_k \times n$.

By Corollary 6 and Theorem 7, we know that $c_2 = \frac{4}{5}$ and $c_3 = \frac{3}{4}$. However, we have yet to determine the value of c_k for any value of k with $k \geq 4$. By Theorem 10, for $k \geq 4$ we know that

$$c_k \leq \frac{k^2 + k + 3}{k^2 + 3k + 1}.$$

In particular, we note that $c_4 \leq \frac{23}{29}$. It is a simple exercise to check that the Cartesian product $K_3 \square K_3$ has perfect Roman domination number 6, implying that $c_4 \geq \frac{2}{3}$. Thus,

$$\frac{2}{3} \leq c_4 \leq \frac{23}{29}.$$

However, the exact value of c_4 is not known.

Problem 2. Characterize the connected cubic graphs that achieve equality in the upper bound of Theorem 7; that is, determine the connected cubic graphs G of order n satisfying $\gamma_R^p(G) = \frac{3}{4}n$.

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Michael A. Henning

Department of Pure and Applied Mathematics
University of Johannesburg
South Africa
E-mail: mahenning@uj.ac.za

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William F. Klostermeyer

School of Computing
University of North Florida
Jacksonville, FL 32224-2669
E-mail: wkloster@unf.edu