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WEAK AND STRONG CONVERGENCE OF AN ITERATIVE METHOD FOR NONEXPANSIVE MAPPINGS IN HILBERT SPACES

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In a real Hilbert space H, starting from an arbitrary initial point $x_0 \in H$, an iterative process is defined as follows: $x_{n+1} = a_n x_n + (1 - a_n) T_f^{\lambda_{n+1}} y_n$, $y_n = b_n x_n + (1 - b_n) T_g^{\beta_n} x_n$, $n \geq 0$, where $T_f^{\lambda_{n+1}} x = Tx - \lambda_{n+1} \mu_f f(Tx)$, $T_g^{\beta_n} x = Tx - \beta_n \mu_g g(Tx)$, $(\forall\ x \in H)$, $T : H \to H$ a nonexpansive mapping with $F(T) \neq \emptyset$ and f (resp. g) $: H \to H$ an η_f (resp. η_g)-strongly monotone and k_f (resp. k_g)-Lipschitzian mapping, $\{a_n\} \subset (0, 1)$, $\{b_n\} \subset (0, 1)$ and $\{\lambda_n\} \subset [0, 1)$, $\{\beta_n\} \subset [0, 1)$. Under some suitable conditions, several convergence results of the sequence $\{x_n\}$ are shown.

1. INTRODUCTION

Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. A mapping $T : H \to H$ is said to be nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for any $x, y \in H$. A mapping $f : H \to H$ is said to be η-strongly monotone if there exists constant $\eta > 0$ such that $\langle fx - fy, x - y \rangle \geq \eta \|x - y\|^2$ for any $x, y \in H$. $f : H \to H$ is said to be k-Lipschitzian if there exists constant $k > 0$ such that $\|fx - fy\| \leq k\|x - y\|$ for any $x, y \in H$.

The interest and importance of construction of fixed points of nonexpansive mappings stem mainly from the fact that it may be applied in many areas, such as image recovery and signal processing (see, e.g., [**1**, **2**, **12**]), solving convex minimization problems (see, e.g., [**3**, **16–19**]). Iterative techniques for approximating fixed points of nonexpansive mappings have been studied by various authors (see, e.g., [**1**, **6–10**, **13**, **14**], etc.), using famous Mann iteration method, Ishikawa iteration method, and many other iteration methods such as, viscosity approximation method [**7**] and CQ method [**8**].

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Let $f : H \to H$ be a nonlinear mapping and K a nonempty closed convex subset of H. The variational inequality problem is formulated as finding a point $u^* \in K$ such that

$$(VI(f,K))\langle f(u^*), \nu - u^*\rangle \geq 0, \quad \forall \nu \in K. \tag{1.1}$$

The variational inequalities were initially studied by Kinderlehrer and Stampacchia [**5**], and ever since have been widely studied. It is well known that the $VI(f,K)$ is equivalent to the fixed point equation

$$u^* = P_K(u^* - \mu f(u^*)), \tag{1.2}$$

where P_K is the projection from H onto K and μ is an arbitrarily fixed constant. In fact, when f is an η-strongly monotone and Lipschitzian mapping on K and $\mu > 0$ small enough, then the mapping defined by the right-hand side of (1.2) is a contraction.

For reducing the complexity of computation caused by the projection P_K, Yamada [**18**] proposed an iteration method to solve the variational inequalities $VI(f,K)$. For arbitrary $u_0 \in H$,

$$u_{n+1} = Tu_n - \lambda_{n+1}\mu f(T(u_n)), n \geq 0, \tag{1.3}$$

where T is a nonexpansive mapping from H into itself, K is the fixed point set of T, f is an η-strongly monotone and k-Lipschitzian mapping on K, $\{\lambda_n\}$ is a real sequence in $[0,1)$, and $0 < \mu < 2\eta/k^2$. Then Yamada [**18**] proved that $\{u_n\}$ converges strongly to the unique solution of the $VI(f,K)$ as $\{\lambda_n\}$ satisfies the following conditions:

$$(1) \lim_{n\to+\infty} \lambda_n = 0; \quad (2) \sum_{n=0}^{+\infty} \lambda_n = \infty; \quad (3) \lim_{n\to+\infty} (\lambda_n - \lambda_{n+1})/\lambda_{n+1}^2 = 0.$$

Based on the idea of iterative process (1.3), recently, Wang [**15**] discussed the more general Mann iteration scheme and gave the following results: Let H be a Hilbert space, $T : H \to H$ a nonexpansive mapping with $F(T) := \{x \in H, Tx = x\} \neq \emptyset$, and $f : H \to H$ an η-strongly monotone and k-Lipschitzian mapping. For any $x_0 \in H$, $\{x_n\}$ is defined by

$$x_{n+1} = a_n x_n + (1 - a_n)T^{\lambda_{n+1}} x_n, \quad n \geq 0, \tag{1.4}$$

where

$$T^{\lambda} x = Tx - \lambda\mu f(Tx), \forall\, x \in H, \tag{1.5}$$

where $\{a_n\} \subset (0,1)$ and $\{\lambda_n\} \subset [0,1)$, then under some suitable conditions, the sequence $\{x_n\}$ is shown to converge strongly to a fixed point of T and the necessary and sufficient conditions that $\{x_n\}$ converges strongly to a fixed point of T are obtained.

Motivated by the above works, we will generalize the scheme (1.4) as follows.

Let H be a Hilbert space, $T : H \to H$ a nonexpansive mapping with $F(T) \neq \emptyset$ and f(resp. g) : $H \to H$ an η_f(resp. η_g)-strongly monotone and k_f(resp. k_g)-Lipschitzian mapping. For any $x_0 \in H$, $\{x_n\}$ is defined by

$$\begin{aligned} x_{n+1} &= a_n x_n + (1-a_n) T_f^{\lambda_{n+1}} y_n, \\ y_n &= b_n x_n + (1-b_n) T_g^{\beta_n} x_n, \quad n \geq 0, \end{aligned} \tag{1.6}$$

where

$$\begin{aligned} T_f^{\lambda_{n+1}} x &= Tx - \lambda_{n+1}\mu_f f(Tx), \quad \forall\, x \in H, \\ T_g^{\beta_n} x &= Tx - \beta_n \mu_g g(Tx), \quad \forall\, x \in H, \end{aligned} \tag{1.7}$$

and $\{a_n\} \subset (0,1)$, $\{b_n\} \subset (0,1)$ and $\{\lambda_n\} \subset [0,1)$, $\{\beta_n\} \subset [0,1)$ satisfy the following conditions:

(i) $\alpha \leq a_n \leq 1-\alpha$, $\beta \leq b_n \leq 1-\beta$ *for some* $\alpha, \beta \in (0, 1/2)$;

(ii) $\sum\limits_{n=1}^{+\infty} \lambda_n < +\infty$, $\sum\limits_{n=1}^{+\infty} \beta_n < +\infty$;

(iii) $0 < \mu_f < 2\eta_f/k_f^2$, $0 < \mu_g < 2\eta_g/k_g^2$.

2. PRELIMINARIES

In this section we will state some useful notations and lemmas.

A Banach space E is said to satisfy Opial's condition if for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ implies that $\limsup\limits_{n\to+\infty} \|x_n - x\| < \limsup\limits_{n\to+\infty} \|x_n - y\|$ for all $y \in E$ with $y \neq x$, where $x_n \rightharpoonup x$ denotes that $\{x_n\}$ converges weakly to x. It is well known that every Hilbert space satisfies Opial's condition.

A mapping T with domain $D(T)$ and range $R(T)$ in E is said to be demiclosed at p; if whenever $\{x_n\}$ is a sequence in $D(T)$ such that $\{x_n\}$ converges weakly to $x^* \in D(T)$ and $\{Tx_n\}$ converges strongly to p, then $Tx^* = p$.

A mapping $T : K \to E$ is said to be demicompact if, for any sequence $\{x_n\}$ in K such that $\|x_n - Tx_n\| \to 0$ $(n \to \infty)$, there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to $x^* \in K$.

Lemma 2.1. [**18**]. *Let $T^\lambda x = Tx - \lambda\mu f(Tx)$, where $T : H \to H$ is a nonexpansive mapping from H into itself and f is an η-strongly monotone and k-Lipschitzian mapping from H into itself. If $0 \leq \lambda < 1$ and $0 < \mu < 2\eta/k^2$, then T^λ is a contraction and satisfies*

$$\|T^\lambda x - T^\lambda y\| \leq (1-\lambda\tau)\|x-y\|, \ \forall\, x, y \in H, \tag{2.1}$$

where $\tau = 1 - \sqrt{1-\mu(2\eta - \mu k^2)}$.

Lemma 2.2. [**14**]. *Let $\{s_n\}$, $\{t_n\}$ be two nonnegative sequence satisfying for some real number $N_0 \geq 1$,*

$$s_{n+1} \leq s_n + t_n \quad \forall n \geq N_0.$$

If $\sum_{n=1}^{+\infty} t_n < +\infty$, then $\lim_{n\to+\infty} s_n$ exists.

Lemma 2.3. [**4**]. *Let K be a nonempty closed convex subset of a real Hilbert space H and T a nonexpansive mapping from K into itself. If T has a fixed point, then $I - T$ is demiclosed at zero, where I is the identity mapping of H.*

3. MAIN RESULTS

First we give the following key lemma.

Lemma 3.1. *For the iterative process* (1.6), *we have*

(1) $\lim_{n\to+\infty} \|x_n - p\|$ *exists for each* $p \in F(T)$;

(2) $\lim_{n\to+\infty} \|x_n - Tx_n\| = 0$.

Proof. At first we recall the well known identity in Hilbert space H: for any $x, y \in H$ and $t \in [0, 1]$,

$$\|tx + (1-t)y\|^2 = t\|x\|^2 + (1-t)\|y\|^2 - t(1-t)\|x-y\|^2. \tag{3.1}$$

For any $p \in F(T)$, from Lemma 2.1, we have

$$\begin{aligned} \|T_g^{\beta_n} x_n - p\| &= \|T_g^{\beta_n} x_n - T_g^{\beta_n} p + T_g^{\beta_n} p - p\| \\ &\leq (1 - \beta_n \tau_g)\|x_n - p\| + \beta_n \mu_g \|g(p)\|, \end{aligned} \tag{3.2}$$

and

$$\begin{aligned} \|T_f^{\lambda_{n+1}} y_n - p\| &= \|T_f^{\lambda_{n+1}} y_n - T_f^{\lambda_{n+1}} p + T_f^{\lambda_{n+1}} p - p\| \\ &\leq \|T_f^{\lambda_{n+1}} y_n - T_f^{\lambda_{n+1}} p\| + \|T_f^{\lambda_{n+1}} p - p\| \\ &\leq (1 - \lambda_{n+1}\tau_f)\|y_n - p\| + \lambda_{n+1}\mu_f \|f(p)\|, \end{aligned} \tag{3.3}$$

where

$$\tau_g = 1 - \sqrt{1 - \mu_g(2\eta_g - \mu_g k_g^2)}, \;\; \tau_f = 1 - \sqrt{1 - \mu_f(2\eta_f - \mu_f k_f^2)}.$$

Furthermore, by the elementary inequality,

$$2ab \leq ta^2 + (1/t)b^2, \text{ for any } a, b \in \mathbb{R}, \; t > 0,$$

we obtain

$$\begin{aligned}\|T_g^{\beta_n}x_n-p\|^2 &\le (1+\frac{\beta_n\tau_g}{1-\beta_n\tau_g})(1-\beta_n\tau_g)^2\|x_n-p\|^2\\ &\qquad\qquad\qquad + (1+\frac{1-\beta_n\tau_g}{\beta_n\tau_g})\beta_n^2\mu_g^2\|g(p)\|^2\\ &= (1-\beta_n\tau_g)\|x_n-p\|^2+\frac{\beta_n\mu_g^2}{\tau_g}\|g(p)\|^2\end{aligned}\tag{3.4}$$

and

$$\|T_f^{\lambda_{n+1}}y_n-p\|^2 \le (1-\lambda_{n+1}\tau_f)\|y_n-p\|^2+\frac{\lambda_{n+1}\mu_f^2}{\tau_f}\|f(p)\|^2.\tag{3.5}$$

From (3.1) and (3.4), it follows

$$\begin{aligned}\|y_n-p\|^2 &= \|b_n(x_n-p)+(1-b_n)(T_g^{\beta_n}x_n-p)\|^2\\ &= b_n\|x_n-p\|^2+(1-b_n)\|T_g^{\beta_n}x_n-p\|^2-b_n(1-b_n)\|T_g^{\beta_n}x_n-x_n\|^2\\ &\le [b_n+(1-b_n)(1-\beta_n\tau_g)]\|x_n-p\|^2+(1-b_n)\frac{\beta_n\mu_g^2}{\tau_g}\|g(p)\|^2\\ &\qquad\qquad\qquad - b_n(1-b_n)\|T_g^{\beta_n}x_n-x_n\|^2.\end{aligned}\tag{3.6}$$

Thus by (3.5) and (3.6), we have

$$\begin{aligned}&|x_{n+1}-p\|^2\\ &= \|a_n(x_n-p)+(1-a_n)(T_f^{\lambda_{n+1}}y_n-p)\|^2\\ &= a_n\|x_n-p\|^2+(1-a_n)\|T_f^{\lambda_{n+1}}y_n-p\|^2-a_n(1-a_n)\|T_f^{\lambda_{n+1}}y_n-x_n\|^2\\ &\le \{a_n+(1-a_n)(1-\lambda_{n+1}\tau_f)[b_n+(1-b_n)(1-\beta_n\tau_g)]\}\|x_n-p\|^2\\ &\quad + (1-a_n)(1-\lambda_{n+1}\tau_f)(1-b_n)\frac{\beta_n\mu_g^2}{\tau_g}\|g(p)\|^2+(1-a_n)\frac{\lambda_{n+1}\mu_f^2}{\tau_f}\|f(p)\|^2\\ &\quad - (1-a_n)(1-\lambda_{n+1}\tau_f)b_n(1-b_n)\|T_g^{\beta_n}x_n-x_n\|^2\\ &\qquad\qquad\qquad - a_n(1-a_n)\|T_f^{\lambda_{n+1}}y_n-x_n\|^2,\end{aligned}\tag{3.7}$$

which implies

$$\|x_{n+1}-p\|^2 \le \|x_n-p\|^2+\frac{\beta_n\mu_g^2}{\tau_g}\|g(p)\|^2+\frac{\lambda_{n+1}\mu_f^2}{\tau_f}\|f(p)\|^2.$$

From Lemma 2.2 and the conditions: $\sum\limits_{n=1}^{+\infty}\lambda_n<+\infty$, $\sum\limits_{n=1}^{+\infty}\beta_n<+\infty$, it follows that $\lim\limits_{n\to+\infty}\|x_n-p\|$ exists for each $q\in F(T)$. It follows that $\{x_n\}$ is bounded. From the iterative process (1.6) we have

$$\|x_{n+1}-x_n\| = (1-a_n)\|T_f^{\lambda_{n+1}}y_n-x_n\|\tag{3.8}$$

and

$$\|y_n - x_n\| = (1 - b_n)\|T_g^{\beta_n} x_n - x_n\|. \tag{3.9}$$

By (3.7), (3.8) and the condition $a_n \in [\alpha, 1-\alpha]$, it follows that

$$\|x_{n+1} - p\|^2 \le \|x_n - p\|^2 + \frac{\beta_n \mu_g^2}{\tau_g} \|g(p)\|^2 + \frac{\lambda_{n+1}\mu_f^2}{\tau_f} \|f(p)\|^2 - \frac{\alpha}{1-\alpha} \|x_{n+1} - x_n\|^2,$$

that is to say that

$$\frac{\alpha}{1-\alpha} \|x_{n+1} - x_n\|^2 \le \|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \frac{\beta_n \mu_g^2}{\tau_g} \|g(p)\|^2 + \frac{\lambda_{n+1}\mu_f^2}{\tau_f} \|f(p)\|^2$$

which implies that

$$\lim_{n\to+\infty} \|x_{n+1} - x_n\| = 0. \tag{3.10}$$

In addition, from (3.8), we know that

$$\lim_{n\to+\infty} \|T_f^{\lambda_{n+1}} y_n - x_n\| = 0. \tag{3.11}$$

From (3.9), (3.7) and similar proof as (3.10) and (3.11), we have

$$\lim_{n\to+\infty} \|y_n - x_n\| = 0 \tag{3.12}$$

and

$$\lim_{n\to+\infty} \|T_g^{\beta_n} x_n - x_n\| = 0. \tag{3.13}$$

Thus

$$\begin{aligned} \|x_n - Tx_n\| &= \|x_n - T_g^{\beta_n} x_n + T_g^{\beta_n} x_n - Tx_n\| \\ &\le \|x_n - T_g^{\beta_n} x_n\| + \beta_n \mu_g \|g(Tx_n)\|. \end{aligned} \tag{3.14}$$

Since $\{x_n\}$ is bounded, then $\{Tx_n\}$ and $\{g(Tx_n)\}$ are bounded as well. Therefore $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. □

Theorem 3.2. *The iterative process $\{x_n\}$, which is taken as in* (1.6), *converges weakly to a fixed point of T.*

Proof. The proof is normal. It follows from Lemma 3.1 that $\lim_{n\to+\infty} \|x_n - p\|$ exists and $\{x_n\}$ is bounded. Now we prove that $\{x_n\}$ has a unique weak subsequential limit in $F(T)$. To prove this, let p_1 and p_2 be weak limits of subsequences $\{x_{n_k}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. It follows from Lemma 2.3 and Lemma 3.1 that $\lim_{n\to+\infty} \|x_n - Tx_n\| = 0$ and $I - T$ is demiclosed with respect to zero, therefore we obtain $Tp_1 = p_1$. Similarly $Tp_2 = p_2$, i.e., $p_1, p_2 \in F(T)$. Next we prove the uniqueness. For this purpose that $p_1 \neq p_2$. then by OPIAL's condition, we obtain

$$\begin{aligned}\lim_{n\to+\infty} \|x_n - p_1\| &= \lim_{k\to+\infty} \|x_{n_k} - p_1\| < \lim_{k\to+\infty} \|x_{n_k} - p_2\| \\ &= \lim_{n\to+\infty} \|x_n - p_2\| = \lim_{j\to+\infty} \|x_{n_j} - p_2\| \\ &< \lim_{j\to+\infty} \|x_{n_j} - p_1\| = \lim_{n\to+\infty} \|x_n - p_1\|.\end{aligned}$$

This is a contradiction. Hence $\{x_n\}$ converges weakly to a point in $F(T)$. □

Theorem 3.3. *Let T be completely continuous, then the iterative process $\{x_n\}$, which is taken as in* (1.6), *converges strongly to a fixed point of T.*

Proof. By Lemma 3.1, $\{x_n\}$ is bounded and $\lim\limits_{n\to+\infty} \|x_n - Tx_n\| = 0$, then $\{Tx_n\}$ is also bounded. Since T is completely continuous, there exists subsequence $\{Tx_{n_j}\}$ of $\{Tx_n\}$ and $p \in H$, such that $\|Tx_{n_j} - p\| \to 0$ as $n_j \to +\infty$. It follows from Lemma 3.1 that $\lim\limits_{n_j\to+\infty} \|x_{n_j} - Tx_{n_j}\| = 0$. So by the continuity of T and Lemma 3.1, we have $\lim\limits_{n_j\to+\infty} \|x_{n_j} - p\| = 0$ and $p \in F(T)$. Furthermore by Lemma 3.1 again, we get that $\lim\limits_{n\to+\infty} \|x_n - p\|$ exists. Thus $\lim\limits_{n\to+\infty} \|x_n - p\| = 0$ which implies the desired result. □

Theorem 3.4. *Let T be demicompact, then the iterative process $\{x_n\}$, which is taken as in* (1.6), *converges strongly to a fixed point of T.*

Proof. Since T is demicompact, $\{x_n\}$ is bounded and $\lim\limits_{n\to+\infty} \|x_n - Tx_n\| = 0$, then there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to $p \in H$. It follows from Lemma 2.3 that $p \in F(T)$. Since the subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges strongly to p and $\lim\limits_{n\to+\infty} \|x_n - p\|$ exists for all $p \in F(T)$ by Lemma 3.1, then $\{x_n\}$ converges strongly to the common fixed point $p \in F(T)$. The proof is completed. □

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