

## BOOK REVIEW

ALBERT W. MARSHALL, INGRAM OLKIN, BARRY C. ARNOLD. Inequalities: theory of majorization and its applications. Second edition. Springer Series in Statistics. Springer, New York, 2009. xxviii+909 pp. ISBN: 978-0-387-40087-7

This is a revised and expanded edition of the book [1] which since 1979 has been an invaluable source of information on results and techniques in Schur convexity (and classical convexity as well), majorization inequalities and related topics, together with their numerous applications in Mathematics and Statistics. Instead of spelling out its long title, we used to simply call it *the Marshall-Olkin's book*; the successful team is now complemented with Barry Arnold.

Compared to the first edition, the volume of the text as well as the reference list has increased over fifty percent. In the preface of [1] the authors wrote: *The book is organized so that it can be used in a variety of ways for a variety of purposes. Sequential reading is not necessary.* This remains true for the second edition as well.

Since the appearance of [1], the amount of published research has at least doubled. The new material is appropriately placed in the body of the text or in supplements at the end of chapters. The new edition preserves the original structure of the book: it is divided into five parts each consisting of several chapters (20 in total). The names of the five parts as well as of the chapters remain the same as in [1], and they are as follows: *Theory of majorization* (Chapters 1-6: Introduction, Doubly stochastic matrices, Schur-convex functions, Equivalent conditions for majorization, Preservation and generation of majorization, Rearrangements and majorization), *Mathematical applications* (Chapters 7-10: Combinatorial analysis, Geometric inequalities, Matrix theory, Numerical analysis), *Stochastic applications* (Chapters 11-13: Stochastic majorizations, Probabilistic, statistical and other applications, Additional statistical applications), *Generalizations* (Chapters 14-15: Orderings extending majorization, Multivariate majorization), and *Complementary topics* (Chapters 16-20: Convex functions and some classical inequalities, Stochastic ordering, Total positivity, Matrix factorizations, compounds, direct products, and M-matrices, and Extremal representations of matrix functions).

Some sections and the subsections of different chapters are altered in order to cover various developments in the field. For example the solution to problem of characterization of doubly stochastic matrices from [1, p.31] is presented in Section 2.D; subsection *Some open problems* [1, p.257] related to the inequality for Schur-Hadamard products is replaced by solutions and a discussions in Section 9.J; subsection *Open questions* in [1, p.269] is replaced by *Matrix That Minimizes the Residual* in Section 10.B.

The authors were concerned with the history of the subject as well. The book contains the biographical appendix with short biographical facts, accompanied with the photographs of the classics: Hugh Dalton, Godfrey Hardy, Johan Jensen, Jovan Karamata, John Littlewood, Max Lorenz, Robert Muirhead, George Pólya and Issai Schur.

Inequalities of Muirhead and Karamata are very important steps in the development of the majorization idea. It is less known, but nonetheless interesting that these two inequalities are very popular amongst the young mathematicians participating in the high school competitions, such as the International Mathematical Olympiad. The reason for this is that many inequalities (in particular the ones appearing at the mathematical competitions) can be derived from these two inequalities. Very interesting articles [2],[3],[4] (not in the reference list of the present book), are written for the high school and university students with the aim to popularize the area of majorization.

This nicely written book of encyclopedic nature, with a precise and coherent exposition, remains to be a compendium to those who wish to study and apply majorization methods to various parts of mathematics.

#### REFERENCES

1. ALBERT W. MARSHALL, INGRAM OLKIN: *Inequalities: Theory of majorization and its applications*. Mathematics in Science and Engineering, 143. Academic Press, Inc. [Harcourt Brace Jovanovich, Publishers], New York-London, xx+569 pp. ISBN: 0-12-473750-1.
2. A. I. HRABROV: *Around Mongolian inequality* (in Russian). *Matematicheskoe prosveshchenie*, **3** (7) (2003), 149–162.
3. D. NOMIROVSKI: *Karamata's Inequality* (in Russian). *Kvant*, **4**, 43–45.
4. ZORAN KADELBURG, DUŠAN ĐUKIĆ, MILIVOJE LUKIĆ, IVAN MATIĆ: *Inequalities of Karamata, Schur and Muirhead, and some applications*. *The Teaching of Mathematics*, **8** (1), (2005), 31–45.

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